EN_51: — The structural degree of $\tilde{n}$ and the non-circularity of G

Víctor Estrada Díaz
UNIHOLOG

Summary

In EN_49, the classes U_c were introduced as families of expressions that determine the same constant.

In EN_50, the structural primality of those classes and the structural order of dependence were proposed:

$$\operatorname{osd}(U_c).$$

This document applies that idea to the case of $\tilde{n}$.

The central question is not whether $\tilde{n}$ can be written as:

$$\tilde{n} = 4l_p^2 .$$

The central question is different:

$$\operatorname{osd}(U_{\tilde{n}}) = 1$$

or:

$$\operatorname{osd}(U_{\tilde{n}}) > 1 .$$

If $\operatorname{osd}(U_{\tilde{n}}) = 1$, then $\tilde{n}$ would be a primary geometric constant.

If $\operatorname{osd}(U_{\tilde{n}}) > 1$, then there must exist a minimal mathematical or geometric relation that determines $\tilde{n}$ from other classes.

1. The constant $\tilde{n}$

In UNIHOLOG, $\tilde{n}$ is not introduced as an arbitrary constant.

It is interpreted as a relation between two geometries.

Therefore, its primary meaning is not dynamical, gravitational or empirical, but geometric.

The expression:

$$\tilde{n} = 4l_p^2$$

indicates an equivalence of value with an area associated with the Planck scale.

But this equality does not by itself determine the logical order of dependence.

The same equality can be read in different directions if the foundation is not declared.

2. Standard reading and UNIHOLOG reading

In the standard reading, one starts from:

$$l_p^2 = \frac{\hbar G}{c^3}.$$

Then:

$$\tilde{n} = 4l_p^2 = \frac{4\hbar G}{c^3}.$$

This reading suggests the chain:

$G \longrightarrow l_p^2 \longrightarrow \tilde{n}$.

But UNIHOLOG proposes another conceptual direction:

relation between two geometries$\longrightarrow \tilde{n} \longrightarrow l_p^2 \longrightarrow G$.

In this second reading, G is not a premise.

G is a consequence.

3. Solving for G

If:

$\tilde{n} = \frac{4\hbar G}{c^3}$,

then:

$G = \frac{\tilde{n} c^3}{4\hbar}$.

This equality shows that, once $\tilde{n}$ is known, the constant G is determined from $\tilde{n}$, c and $\hbar$.

Therefore, within this reading:

$U_G \subseteq \mathrm{Gen}\,(U_{\tilde{n}}, U_c, U_\hbar)$.

Thus:

U_G

is not a primary class with respect to $U_{\tilde{n}}$.

It is a derived class.

4. The objection of circularity

The usual objection can be formulated as follows:

1. $\tilde{n}$ is written as $4l_p^2$.
2. l_p is defined through G.
3. Therefore, $\tilde{n}$ would depend on G.

This objection is valid only if G is taken as the starting point.

But if $\tilde{n}$ is obtained independently as a geometric relation, then G does not participate in the primary definition of $\tilde{n}$.

In that case:

$\tilde{n}$

does not depend on:

G.

On the contrary:

G

is determined by:

$\tilde{n}$.

Therefore, there is no circularity.

There is an inversion of foundation.

5. Proposition of non-circularity

Proposition 1

If $\tilde{n}$ is obtained through a geometric relation independent of G, and if afterwards one defines:

$G = \frac{\tilde{n}c^3}{4\hbar}$,

then the formal appearance of l_p in:

$\tilde{n} = 4l_p^2$

does not constitute a circular definition of $\tilde{n}$.

Proof

A circular definition would require $\tilde{n}$ to depend on G and G to depend on $\tilde{n}$ within the same definitional cycle.

But by hypothesis, $\tilde{n}$ is obtained first, through a geometric relation independent of G.

Afterwards, G is calculated as:

$G = \frac{\tilde{n}c^3}{4\hbar}$.

Therefore, G is not a premise of $\tilde{n}$.

It is a consequence of $\tilde{n}$.

Hence there is no definitional cycle.

This proves the claim.

6. The question about the degree of $\tilde{n}$

Although non-circularity is resolved under the geometric hypothesis, a deeper question remains:

$\operatorname{osd}(U_{\tilde{n}}) = \ ?$

This question does not refer to G.

It refers to the internal structure of $U_{\tilde{n}}$.

It may be the case that:

$\operatorname{osd}(U_{\tilde{n}}) = 1$.

In that case, $\tilde{n}$ would be a primary geometric class.

It may also be the case that:

$$\operatorname{osd}(U_{\tilde{n}}) > 1 .$$

In that case, $\tilde{n}$ would not be an isolated primary class, but the result of a minimal relation between several classes.

7. Primary case

If:

$$\operatorname{osd}(U_{\tilde{n}}) = 1,$$

then $\tilde{n}$ depends only on its own original geometric relation.

There would be no non-trivial combination of other constants determining it in a more fundamental way.

The class:

$U_{\tilde{n}}$

would be analogous, in a structural sense, to a prime class.

This does not mean that $U_{\tilde{n}}$ has only one expression.

It may contain limits, series, integrals or equivalent constructions.

What it means is that none of those expressions reveals a more basic generative dependence on other classes.

8. Composite case

If:

$$\operatorname{osd}(U_{\tilde{n}}) > 1,$$

then there must exist a minimal relation:

$$R(\tilde{n}, c_1, c_2, \ldots, c_k) = 0$$

such that none of the participating classes can be removed without destroying the relation.

In that case:

$$U_{\tilde{n}}$$

would be a composite class.

But not composite through G.

Composite through a deeper mathematical or geometric relation that remains to be found.

9. Analogy with Euler's identity

Euler's identity:

$$e^{i\pi} + 1 = 0$$

relates:

$$U_e, \quad U_i, \quad U_\pi, \quad U_1, \quad U_0 .$$

It does not say that e is π.

It does not say that $U_e = U_\pi$.

What it shows is that there exist closed relations where several classes appear necessarily.

If one tries to solve for one of them, the others remain as structural support.

Analogously, if $\tilde{n}$ is not primary, there must exist a closed relation:

$R(\tilde{n}, c_1, \ldots, c_k) = 0$

where $\tilde{n}$ appears as a necessary part of a broader structure.

10. Search criterion

The search for $\tilde{n}$ must not be confused with a numerological search.

It is not enough to find an expression that approximates:

$4l_p^2$.

The sought expression must at least:

1. determine exactly $\tilde{n}$ or propose an explicit conjecture;
2. not use G as a premise;
3. have dimension or interpretation compatible with a geometric area;
4. arise from a non-trivial relation;
5. admit a clear geometric reading;
6. reduce or explain the structural degree of $U_{\tilde{n}}$.

If an expression only rewrites:

$\tilde{n} = \frac{4\hbar G}{c^3}$,

then it does not resolve the question.

It only translates $\tilde{n}$ into the standard language of Planck units.

11. Formulation of the open problem

The open problem can be formulated as:

Determine whether $U_{\tilde{n}}$ is primary or composite.

Equivalently:

$\text{osd}\,(U_{\tilde{n}}) = 1 \quad \text{or} \quad \text{osd}\,(U_{\tilde{n}}) > 1$.

If:

$$\text{osd}\,(U_{\tilde{n}}) = 1,$$

then $\tilde{n}$ is a primary geometric constant.

If:

$$\text{osd}\,(U_{\tilde{n}}) > 1,$$

then there exists a minimal relation:

$$R(\tilde{n}, c_1, \ldots, c_k) = 0$$

that still has to be identified.

12. Conclusion

$\tilde{n}$ must not be interpreted as a simple consequence of G.

In UNIHOLOG, $\tilde{n}$ proceeds from a relation between two geometries.

For that reason, the correct conceptual chain is:

$$\text{geometry} \longrightarrow \tilde{n} \longrightarrow G\,.$$

This destroys the objection of circularity provided that the geometric derivation of $\tilde{n}$ is independent of G.

The question that remains open is deeper:

$$\text{osd}\,(U_{\tilde{n}}) = \,?$$

If the degree is one, $\tilde{n}$ will be a prime geometric class.

If the degree is greater than one, then someday the minimal mathematical or geometric relation from which it derives must be found.