EN_50: — Structural primality of the classes U_c and the degree of $\tilde{n}$

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Abstract

In EN_49 the idea of a class U_c was introduced as the set of expressions that determine the same constant c.

This document continues that construction and asks a deeper question: if a constant is represented by a class U_c, can that class be considered primary, or does it structurally depend on other classes?

The question is formulated through a notion of structural primality and through the structural order of dependence:

$\text{osd}\,(U_c)$.

The central case is the geometric constant $\tilde{n}$, defined in the UNIHOLOG framework as a relation between two geometries. Although it can be written as

$\tilde{n} = 4l_p^2$,

its meaning does not come from G, but from a prior geometric relation. Therefore, the question is not whether $\tilde{n}$ can be written using l_p, but whether $U_{\tilde{n}}$ is a prime class or a composite class.

1. Starting point

Let c be a constant.

We define U_c as the set of expressions that determine exactly the same value c:

$$U_c = \{A\text{: valor } (A) = c\} .$$

Two expressions A and B belong to the same class U_c if both determine the same value:

$$A \sim_c B \Leftrightarrow \text{ valor } (A) = \text{ valor } (B) = c .$$

This definition does not identify the procedures with each other. An exact algebraic expression, a sequence, an integral, a derivative or a continued fraction may belong to the same class if they determine the same value.

For example:

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

and

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

are different expressions, but both belong to U_e.

2. Primary expressions and derived expressions

Not all expressions in a class U_c have the same structural value.

If an expression is obtained by a trivial transformation of another already known expression, it does not open a new path toward the constant.

For example, if

$$A \in U_c ,$$

then also:

$$A + 0 \in U_c ,$$

$$A \cdot 1 \in U_c ,$$

$\frac{2A}{2} \in U_c$.

These expressions formally belong to U_c, but they are not primary.

We shall call a primary expression of U_c an expression that determines c and is not presented as a simple trivial transformation of another expression already included in the catalogue.

This notion is relative to available knowledge. An expression may appear primary until a derivation is found that connects it with an earlier expression.

Thus, provisionally:

$U_c^{\text{prim}} = \{A \in U_c : A \text{ is not a trivial derivative of another known expression of } U_c\}$.

3. Generation between classes

Let B be a family of classes:

$B = \{U_{c_1}, U_{c_2}, \ldots, U_{c_k}\}$.

We shall say that a constant c is generable from B if there exists an admissible function F such that:

$c = F(c_1, c_2, \ldots, c_k)$.

In that case:

$U_c \subseteq \operatorname{Gen}(B)$.

Here $\operatorname{Gen}(B)$ represents the set of classes obtainable from B through admitted operations.

These operations may include sum, product, quotient, powers, roots, limits, series, infinite products, integrals, derivatives or special functions, provided that they are well defined and justified.

For example, if:

$A \in U_\alpha$

and

$B \in U_{\beta}$,

then:

$A + B \in U_{\alpha + \beta}$,

$AB \in U_{\alpha\beta}$,

and, if $\beta \neq 0$:

$\frac{A}{B} \in U_{\alpha / \beta}$.

4. Structural primality

The analogy with prime numbers appears when we ask whether a class can be obtained from others.

A prime number does not decompose as a nontrivial product of smaller integers.

Analogously, a class U_c may be called structurally prime if it is not obtained as a nontrivial combination of other independent classes.

We define:

U_c is prime with respect to B

if:

$c \notin \text{Gen}(B \setminus \{U_c\})$.

This does not mean that U_c lacks internal structure.

A prime class may contain many expressions:

$A_1, A_2, A_3, \ldots \in U_c$.

What is denied is not the internal multiplicity of expressions, but the external dependence on other classes.

5. Internal development through N

Every computable constant admits at least one representation through a sequence indexed by natural numbers.

That is, for a computable constant c, there exists a rational or real sequence:

$a:N \to Q$

such that:

$$\lim_{n \to \infty} a_n = c .$$

Therefore:

$$\lim_{n \to \infty} a_n \in U_c .$$

This shows that a class may be structurally prime and, at the same time, admit infinite internal developments.

Primality does not exclude development.

It only excludes nontrivial generative dependence on other classes.

6. Structural order of dependence

We define the structural order of dependence of a class U_c as:

$$\operatorname{osd}(U_c) .$$

Informally, $\operatorname{osd}(U_c)$ measures the minimum number of classes necessary to determine U_c through a closed structural relation.

One possible formulation is:

$$\operatorname{osd}(U_c) = \min \{k : \exists R(c, c_1, \ldots, c_k) = 0\},$$

where the relation R must be nontrivial and must not be reducible by eliminating one of the participating classes.

If:

$$\operatorname{osd}(U_c) = 1,$$

then U_c behaves as a primary class.

If:

$$\operatorname{osd}(U_c) > 1,$$

then U_c appears as part of a minimal relation with other classes.

7. Euler's identity

Euler's identity is:

$$e^{i\pi} + 1 = 0\,.$$

This relation connects the classes:

$$U_e\,, \quad U_i\,, \quad U_\pi\,, \quad U_1\,, \quad U_0\,.$$

It does not state that:

$$U_e = U_\pi\,.$$

Nor does it state that e and π are the same constant.

What it shows is that there exists a closed structural relation in which these classes appear simultaneously.

If one of them is taken as unknown, the others are necessary to reconstruct it within that relation.

For example, formally:

$$e^{i\pi} = -1\,.$$

If one tries to solve for π, one obtains:

$$\pi = \frac{\log(-1)}{i},$$

but this expression already requires the complex structure, the unit, zero, the exponential and a choice of branch of the logarithm.

Therefore, Euler does not eliminate the independence of e, π or i. It shows a minimal relation where none of those classes can be removed without changing the structure.

8. Independence and relation are not the same

Two classes may be independent and still be related.

Generative independence states that one class is not obtained from another through the admitted rules:

$U_a \nsubseteq \operatorname{Gen}(U_b)$.

But a relation may exist:

$R(a, b) = 0$.

Therefore, one must distinguish:

generative dependence

from:

structural relation.

Euler's identity is precisely an example of a deep structural relation between classes that do not reduce trivially to one another.

9. The case of $\tilde{n}$

In UNIHOLOG, the constant $\tilde{n}$ is presented as a geometric constant.

It can be written:

$\tilde{n} = 4l_p^2$.

But this writing must not necessarily be interpreted in the direction:

$$G \longrightarrow l_p \longrightarrow \tilde{n}\,.$$

In the UNIHOLOG framework, the conceptual direction is:

$$\text{relation between two geometries} \longrightarrow \tilde{n} \longrightarrow l_p^2 \longrightarrow G\,.$$

That is, $\tilde{n}$ is not introduced as a constant derived from G, but as a prior geometric relation.

Then, if one uses:

$$l_p^2 = \frac{\hbar G}{c^3},$$

one obtains:

$$\tilde{n} = \frac{4\hbar G}{c^3}\,.$$

And therefore:

$$G = \frac{\tilde{n}c^3}{4\hbar}\,.$$

Thus, in this framework, G is not primary with respect to $\tilde{n}$.

On the contrary:

$$U_G \subseteq \text{Gen}\,(U_{\tilde{n}}, U_c, U_\hbar)\,.$$

Therefore:

U_G is composite with respect to $U_{\tilde{n}}$.

10. Anticircularity of $\tilde{n}$

The circularity objection says:

$$\tilde{n} = 4l_p^2$$

and:

$$l_p^2 = \frac{\hbar G}{c^3}.$$

Therefore, it would seem that $\tilde{n}$ depends on G.

But this objection is valid only if G is taken as a premise.

If $\tilde{n}$ is obtained independently as a geometric relation between two structures, then G is not a premise but a consequence:

$$G = \frac{\tilde{n} c^3}{4\hbar}.$$

The same equality can be read in two directions.

The standard reading is:

$$G \rightarrow l_p^2 \rightarrow \tilde{n}.$$

The UNIHOLOG reading is:

$$\tilde{n} \rightarrow l_p^2 \rightarrow G.$$

Therefore, there is no circularity if the derivation of $\tilde{n}$ is geometric and independent of G.

11. The open question

We do not yet know the structural degree of $\tilde{n}$.

The question is:

$$\operatorname{osd}(U_{\tilde{n}}) = 1$$

or:

$$\operatorname{osd}(U_{\tilde{n}}) > 1.$$

If:

$$\operatorname{osd}(U_{\tilde{n}}) = 1,$$

then $U_{\tilde{n}}$ would be a primary class.

In that case, $\tilde{n}$ would be a geometric value that does not depend on other classes in order to be determined.

If:

$$\operatorname{osd}(U_{\tilde{n}}) > 1,$$

then there must exist a minimal mathematical or geometric relation:

$$R(\tilde{n}, c_1, c_2, \ldots, c_k) = 0 .$$

In that case, $\tilde{n}$ would not be an isolated primary constant, but a constant emerging from a deeper structural relation.

12. Conceptual consequence

The search for $\tilde{n}$ does not consist in looking for any arbitrary numerical combination.

The correct search is:

$$\exists R : R(\tilde{n}, c_1, \ldots, c_k) = 0$$

where R is:

- nontrivial;
- minimal;
- geometrically interpretable;
- independent of G as a premise;
- capable of determining $\tilde{n}$.

If such a relation is found, then the structural order of dependence of $U_{\tilde{n}}$ will have been identified.

If no such relation exists and $\tilde{n}$ depends only on itself as a primary geometric relation, then:

$$\operatorname{osd}(U_{\tilde{n}}) = 1 .$$

13. Conclusion

EN_49 made it possible to define classes of expressions U_c.

EN_50 introduces a new question:

which classes are structurally prime?

Euler's identity shows that several classes can be related in a minimal structure without reducing trivially to one another.

The case of $\tilde{n}$ raises an analogous question:

$$\text{osd}\,(U_{\tilde{n}}) = 1 \quad \text{or} \quad \text{osd}\,(U_{\tilde{n}}) > 1$$

If $U_{\tilde{n}}$ is prime, then $\tilde{n}$ expresses an irreducible geometric constant.

If it is not prime, then one day a minimal mathematical or geometric relation that determines it must be found.

In both cases, G is placed as a class derived with respect to $\tilde{n}$, not as its foundation.